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Non-Zero θ_{13} and δ_{CP} in a Neutrino Mass Model with A_4 Symmetry

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Abstract: In this paper, we consider a neutrino mass model based on A_4 symmetry. The spontaneous symmetry breaking in this model is chosen to obtain tribimaximal mixing in the neutrino sector. We introduce $Z_2 \times Z_2$ invariant perturbations in this model which can give rise to acceptable values of θ_{13} and δ_{CP}. Perturbation in the charged lepton sector alone can lead to viable values of θ_{13}, but cannot generate δ_{CP}. Perturbation in the neutrino sector alone can lead to acceptable θ_{13} and maximal CP violation. By adjusting the magnitudes of perturbations in both sectors, it is possible to obtain any value of δ_{CP}.

Contents

1 Introduction

The discovery of neutrino oscillations has triggered a lot of experimental and theoretical effort to understand the physics of lepton masses and mixing. Since flavor mixing occurs due to the mismatch between the mass and flavor eigenstates, neutrinos need to have small non-degenerate masses [1, 2]. During the past two decades, many neutrino oscillation experiments have been performed and the values of oscillation parameters are determined to a very good precision [3–5].

Neutrino oscillation probabilities depend only on the mass-squared differences and the mixing angles. Hence these parameters are determined in the neutrino oscillation experiments. The experimental data has shown two large mixing angles and one small mixing angle. This pattern is different from the case of quark mixing where all angles are small and the mixing matrix is close to identity. The lepton mixing matrix, called Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, is approximately equal to the tribimaximal (TBM) ansatz proposed in ref. [6]. In this ansatz, the mixing angles have values $\tan^2\theta_{12} = \frac{1}{2}$, $\theta_{13} = 0°$, and $\sin^2\theta_{23} = \frac{1}{2}$.

The TBM form of the PMNS matrix is

$$U_{PMNS} \simeq \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \equiv U_{\text{TBM}}, \tag{1.1}$$

where $|U_{e3}| = \sin\theta_{13} = 0$.

Many recent experiments [7–9] have shown that the value of θ_{13} is not only non-zero but is relatively high [10]. The values of other mixing angles also have small deviations from the TBM values. Since θ_{13} is non-zero, the possibility of a CP violating phase (δ_{CP}) in the lepton mixing matrix must be considered seriously. The neutrino oscillation experiments have found two mass-squared differences with very different magnitudes. The smaller mass-squared difference, denoted $\Delta m^2_{21} = m^2_2 - m^2_1$, is positive and is of the order of 10^{-5}eV^2.

The larger mass-squared difference, $\Delta m^2_{31} = m^2_3 - m^2_1$, is of the order of 10^{-3}eV^2, but its sign is not known. This leads to two possible mass hierarchies for neutrinos: normal hierarchy (NH) in which Δm^2_{31} is positive and $m_1 < m_2 < m_3$ and inverted hierarchy (IH) where Δm^2_{31} is negative and $m_3 < m_1 < m_2$. Finding the sign of Δm^2_{31} is a major goal in many experiments like INO [11, 12], ICECube-PINGU [13, 14], and long baseline experiments [15, 16]. The values of mixing angles and mass-squared differences from the global analysis of data is summarized in Table 1 [17].

Parameter	$\text{mean}^{(+1\sigma,+2\sigma,+3\sigma)}_{(-1\sigma,-2\sigma,-3\sigma)}$
$\Delta m^2_{21}[10^{-5}eV^2]$	$7.60^{(+0.19,+0.39,+0.58)}_{(-0.18,-0.34,-0.49)}$
$\Delta m^2_{31}[10^{-3}eV^2]$	$_{(\text{NH})}2.48^{(+0.05,+0.10,+0.16)}_{(-0.06,-0.12,-0.18)}$, $_{(\text{IH})}-2.38^{(+0.05,+0.10,+0.16)}_{(-0.06,-0.12,-0.18)}$
$\sin^2\theta_{12}$	$0.323^{(+0.016,+0.034,+0.052)}_{(-0.016,-0.031,-0.045)}$
$\sin^2\theta_{23}$	$_{(\text{NH})}0.567^{(+0.022,+0.047,+0.067)}_{(-0.128,-0.154,-0.175)}$, $_{(\text{IH})}0.573^{(+0.025,+0.048,+0.067)}_{(-0.043,-0.141,-0.170)}$
$\sin^2\theta_{13}$	$_{(\text{NH})}\ 0.0234^{(+0.002,+0.004,+0.006)}_{(-0.002,-0.0039,-0.0057)}$, $_{(\text{IH})}0.0240^{(+0.0019,+0.0038,+0.0057)}_{(-0.0019,-0.0038,-0.0057)}$

Table 1: The values of mass-squared differences and mixing angles from the global fits [18]. The numbers in the parenthesis are upper/lower uncertainties at (1σ, 2σ, 3σ) confidence level.

To accommodate the small masses of neutrinos in comparison to charged leptons and quarks, a novel mechanism involving Majorana nature of neutrinos, called seesaw mechanism, was introduced in [19–22]. In this mechanism, the right handed partners of neutrinos are introduced with Majorana masses at high scale. The neutrinos, in addition, have Dirac masses of the order of charged lepton masses. The most general neutrino mass matrix is a 6×6 matrix in the space of three left-handed and three right-handed neutrino fields. A diagonalization of this matrix leads to the generation of small Majorana masses for left-handed neutrinos. A common approach to obtain the observed mixing pattern is to constrain the structure of interaction Lagrangian, which gives rise to the mass matrix, using a discrete non-abelian flavor symmetry [23–31]. Many such models are constructed using discrete, non-abelian groups like A_4 [29, 32–35] and S_4 [36–38]. In particular, it was shown

in [33–35] that models based on A_4 symmetry can lead to the prediction of tribimaximal mixing. Being the smallest group with an irreducible triplet representation, A_4 has been popular group for neutrino mass models since its introduction in ref. [32].

In the wake of θ_{13} measurement, it is necessary to modify the models predicting TBM pattern [39–42]. Two major approaches to incorporate the necessary modifications are vacuum misalignment and symmetry breaking via perturbation terms. All models based on discrete symmetry groups require a special vacuum alignment condition to obtain tribimaximal mixing. A deviation from this, that is , a vacuum misalignment can lead to deviations from TBM pattern [43, 44]. Another way to generate deviations from TBM pattern is to add symmetry breaking terms which break the symmetry completely or partially [35, 45]. It is common to have different residual symmetries in charged lepton and neutrino sectors after such a perturbation.

In this paper, we will consider modifications of a model based on A_4 group proposed in [35]. TBM pattern is obtained in this model by breaking A_4 symmetry spontaneously to Z_3 in the charged lepton sector and to Z_2 in the neutrino sector. We first introduce a $Z_2 \times Z_2$ invariant complex perturbation in the charged lepton sector only. This perturbation leads to non-zero value for θ_{13}, small deviations in the values of θ_{12} and θ_{23}, but does not lead to any CP violation. If a real $Z_2 \times Z_2$ perturbation is introduced in the neutrino sector only, viable values of θ_{13} and maximal CP violation are obtained. By introducing perturbations in both the charged lepton and the neutrino sectors, it is possible to obtain any value of δ_{CP} by adjusting their relative strengths.

2 The A_4 Model

A_4 is the group of even permutations on four elements and is the smallest group with a three dimensional irreducible representation which makes it a popular group in neutrino mass modelling. This group has three 1-dimensional irreducible representations and one 3-dimensional irreducible representation.There are two popular approaches to study the three dimensional irreducible representation: the Ma-Rajasekaran (M-R) approach [32] which makes all the $Z_2 \times Z_2$ elements diagonal and the Altarelli-Feruglio (A-F) approach [34] in which the Z_3 elements are diagonal. We will use the M-R convention in our discussion. A_4 has four classes denoted by C1, C2, C3, and C4. The 3×3 matrix representations of the

A_4 elements in each of these classes are:

$$\begin{aligned}
\text{C1}: &\quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\
\text{C2}: &\quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\
\text{C3}: &\quad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}, \\
\text{C4}: &\quad \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.
\end{aligned} \tag{2.1}$$

The Z_3 elements in this group are

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \tag{2.2}$$

The $Z_2 \times Z_2$ elements in this group are

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.3}$$

In this section, we will discuss the details of a type-I seesaw model based on A_4 group proposed in ref.[35]. We limit ourselves to the leptonic sector of the model. The fields in this sector are the three left-handed $SU(2)$ gauge doublets, three right-handed charged-lepton gauge singlets, and three right-handed neutrino gauge singlets. They are assigned to various irreducible representations of the A_4 group. In addition, there are four Higgs doublets, ϕ_i (i = 1,2,3) and ϕ_0, and three scalar singlets χ_i (i=1,2,3). The assignments of the fields under various groups, are given in Table 2.

By using the Clebsh-Gordon decomposition of A_4 tensor products, the complete $G_{SM} \otimes A_4$ invariant (G_{SM} is the standard model gauge symmetry) Yukawa Lagrangian for the leptonic sector can be written as [46]

$$\mathcal{L}_{\text{Yukawa}} = \mathcal{L}_{\text{CL Dirac}} + \mathcal{L}_{\text{N Dirac}} + \mathcal{L}_{\text{N Majorana}}. \tag{2.4}$$

The individual terms of this equation are given by

$$\begin{aligned}
\mathcal{L}_{\text{CL Dirac}} = \; & -\big[h_1(\bar{D}_{1L}\phi_1 + \bar{D}_{2L}\phi_2 + \bar{D}_{3L}\phi_3)l_{1R} \\
& +h_2(\bar{D}_{1L}\phi_1 + \omega^2\bar{D}_{2L}\phi_2 + \omega\bar{D}_{3L}\phi_3)l_{2R} \\
& +h_3(\bar{D}_{1L}\phi_1 + \omega\bar{D}_{2L}\phi_2 + \omega^2\bar{D}_{3L}\phi_3)l_{3R}\big] + \text{h.c.},
\end{aligned} \tag{2.5}$$

	$SU(2)$	$U(1)$	A_4	
D_{iL}	$\underline{\frac{1}{2}}$	Y=-1	$\underline{3}$	left-handed doublets
l_{iR}	$\underline{0}$	Y=-2	$\underline{1} \oplus \underline{1}' \oplus \underline{1}''$	right-handed charged lepton singlets
ν_{iR}	$\underline{0}$	Y= 0	$\underline{3}$	right-handed neutrino singlets
ϕ_i	$\underline{\frac{1}{2}}$	Y= 1	$\underline{3}$	Higgs doublet
ϕ_0	$\underline{\frac{1}{2}}$	Y= 1	$\underline{1}$	Higgs doublet
χ_i	$\underline{0}$	Y= 0	$\underline{3}$	real gauge singlet

Table 2: Assignments of lepton and scalar fields to various irreps of $SU(2)$, $U(1)$, and A_4.

where ω is the cube root of unity,

$$\mathcal{L}_{\text{N Dirac}} = -h_0(\bar{D}_{1L}\nu_{1R} + \bar{D}_{2L}\nu_{2R} + \bar{D}_{3L}\nu_{3R})\tilde{\phi}_0 + \text{h.c.}, \tag{2.6}$$

and

$$\begin{aligned} \mathcal{L}_{\text{N Majorana}} = \; & -\frac{1}{2}\left[M(\nu_{1R}^T C^{-1}\nu_{1R} + \nu_{2R}^T C^{-1}\nu_{2R} + \nu_{3R}^T C^{-1}\nu_{3R})\right] + \text{h.c.}] \\ & -\frac{1}{2}\left[h_\chi(\chi_1(\nu_{2R}^T C^{-1}\nu_{3R} + \nu_{3R}^T C^{-1}\nu_{2R})\right. \\ & +\chi_2(\nu_{3R}^T C^{-1}\nu_{1R} + \nu_{1R}^T C^{-1}\nu_{3R}) \\ & \left.+\chi_3(\nu_{1R}^T C^{-1}\nu_{2R} + \nu_{2R}^T C^{-1}\nu_{1R})\right], \end{aligned} \tag{2.7}$$

where C is the charge conjugation matrix. Here, $\mathcal{L}_{\text{CL Dirac}}$ contributes to the Dirac mass matrix in the charged lepton sector, $\mathcal{L}_{\text{N Dirac}}$ contributes to the Dirac mass matrix in the neutrino sector and $\mathcal{L}_{\text{N Majorana}}$ contributes to the Majorana mass matrix of the right handed neutrinos. $\mathcal{L}_{\text{Yukawa}}$ has an additional $U(1)_X$ symmetry [35]. Under this symmetry the fields D_{iL}, l_{iR} and ϕ_i have quantum numbers $X = 1$ and all other fields have $X = 0$. This symmetry forbids the Yukawa terms of the form $\bar{D}_L\nu_R\tilde{\phi}_i$. These terms are invariant under $G_{SM} \times A_4$ and contribute to the Dirac mass matrix of the neutrinos. Without the contribution of these terms, this matrix retains the simple form needed to obtain the tribimaximal mixing.

Spontaneous symmetry breaking leads to the following scalar VEVs: v_i for ϕ_i, w_i for χ_i, and v_0 for ϕ_0. With these VEVs, we obtain the different mass terms to be

$$-\bar{l}_L M_l^0 l_R - \bar{\nu}_L M_D \nu_R + \frac{1}{2}\nu_R^T C^{-1} M_R \nu_R + h.c., \tag{2.8}$$

where

$$M_l^0 = \begin{pmatrix} h_1v_1 & h_2v_1 & h_3v_1 \\ h_1v_2 & h_2v_2\omega^2 & h_3v_2\omega \\ h_1v_3 & h_2v_3\omega & h_3v_3\omega^2 \end{pmatrix}, \; M_R = \begin{pmatrix} M & h_\chi w_3 & h_\chi w_2 \\ h_\chi w_3 & M & h_\chi w_1 \\ h_\chi w_2 & h_\chi w_1 & M \end{pmatrix}, \tag{2.9}$$

and $M_D = h_0 v_0 I$. Tribimaximal mixing requires a special vacuum alignment given by

$$v_1 = v_2 = v_3 = v, \; w_1 = w_3 = 0, \text{and } h_\chi w_2 = M^{'}. \tag{2.10}$$

The charged lepton mass matrix M_l^0 can be put in a diagonal form by the transformation

$$U_\omega M_l^0 I = \begin{pmatrix} \sqrt{3}vh_1 & 0 & 0 \\ 0 & \sqrt{3}vh_2 & 0 \\ 0 & 0 & \sqrt{3}vh_3 \end{pmatrix} \quad \text{where } U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}. \tag{2.11}$$

The Majorana mass matrix M_R is diagonalized by an orthogonal transformation

$$U_\nu M_R U_\nu^\dagger = \begin{pmatrix} M+M' & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M-M' \end{pmatrix} \qquad \text{where } U_\nu = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{2.12}$$

The PMNS matrix is now obtained to be tribimaximal up to phases on both sides.

$$U = U_\omega U_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -i \end{pmatrix}. \tag{2.13}$$

The vacuum alignment for scalar fields spontaneously breaks the A_4 symmetry in the charged lepton sector (coupling only with ϕ_i) to Z_3 subgroup. In the neutrino sector (coupling with χ and ϕ_0), the residual symmetry is Z_2. The Lagrangian lacks a common symmetry as there is no subgroup between Z_2 and Z_3. A novel feature of the model is that the diagonalizing matrix is completely determined by the symmetry, but the lepton masses are given by the arbitrary coupling constants h_i $(i = 0, 1, 2, 3)$. The seesaw mechanism generates small masses for the left handed neutrinos given by $M_D^T M_R^{-1} M_D$. The masses of the left-handed neutrinos then become $m_D^2/(M+M')$,m_D^2/M, and $m_D^2/(M-M')$, satisfying the relation $2m_2^{-1} = m_3^{-1} + m_1^{-1}$ [47]. For $M' \ll M$, a quasi degenerate spectrum is obtained.

3 Perturbation in Charged Lepton Sector

In the model discussed till now, the PMNS matrix has the tribimaximal form with zero θ_{13} and no CP violation. To generate non-zero values for these, we add small perturbations to the above model. We first introduce a symmetry breaking term in the charged lepton sector which is invariant under the subgroup $Z_2 \times Z_2$. In order to construct such a perturbation, it is required to know the breaking pattern of A_4 irreducible representations into $Z_2 \times Z_2$ irreducible representations. The group $Z_2 \times Z_2$ is the normal subgroup of A_4 with four elements. It has one trivial singlet representation $\underline{\hat{\mathbf{1}}}(1,1,1,1)$ and three non-trivial singlet representations, viz. $\underline{\hat{\mathbf{1}}}'''(1,1,-1,-1)$, $\underline{\hat{\mathbf{1}}}''(1,-1,1,-1)$ and $\underline{\hat{\mathbf{1}}}'(1,-1,-1,1)$. The breaking of A_4 triplet into $Z_2 \times Z_2$ irreducible representations can be readout from the diagonal matrix elements of $Z_2 \times Z_2$ in the M-R basis, shown in eq.(2.3). This is given as

$$\begin{aligned} (\underline{\mathbf{3}}) \text{ of } A_4 &\xrightarrow{\text{breaks into}} (\underline{\hat{\mathbf{1}}}''' \oplus \underline{\hat{\mathbf{1}}}'' \oplus \underline{\hat{\mathbf{1}}}') \text{ of } Z_2 \times Z_2 \\ (\underline{\mathbf{1}}, \underline{\mathbf{1}}', \underline{\mathbf{1}}'') \text{ of } A_4 &\xrightarrow{\text{breaks into}} \qquad (\underline{\hat{\mathbf{1}}}) \qquad \text{ of } Z_2 \times Z_2. \end{aligned} \tag{3.1}$$

The general $Z_2 \times Z_2$ invariant perturbation can be written as

$$\underset{(\mathbf{3})}{h_1 \bar{\mathrm{D}}_{\mathrm{L}}} M_1 \underset{(\mathbf{3})}{\phi} \underset{(\hat{\underline{\mathbf{1}}})}{l_{1\mathrm{R}}} + \underset{(\mathbf{3})}{h_2 \bar{\mathrm{D}}_{\mathrm{L}}} M_2 \underset{(\mathbf{3})}{\phi} \underset{(\hat{\underline{\mathbf{1}}})}{l_{2\mathrm{R}}} + \underset{(\mathbf{3})}{h_3 \bar{\mathrm{D}}_{\mathrm{L}}} M_3 \underset{(\mathbf{3})}{\phi} \underset{(\hat{\underline{\mathbf{1}}})}{l_{3\mathrm{R}}} \tag{3.2}$$

where $\bar{D}_L$, ϕ are the three-dimensional reducible representations of $Z_2 \times Z_2$ and l_R's are trivial singlets. For the perturbation to be $Z_2 \times Z_2$ invariant , the matrices M_1, M_2 and M_3 must commute with the matrices given in eq. (2.3). This is satisfied by any diagonal matrix.

It can be observed that introducing a multiplicative factor in the i^{th} row of charged lepton mass matrix in eq. (2.9) will introduce a reciprocal factor in the i^{th} column of its diagonalizing matrix U_ω. The U_{e3} element of the PMNS matrix in the TBM form is zero because the 11 and 13 elements of U_ω are equal. The perturbation terms in eq. (3.2) can disturb this balance and lead to non-zero U_{e3}. The value of U_{e3} (and hence θ_{13}) depends on the elements of M_1, M_2, and M_3. In order to obtain a simple form for the perturbed charged lepton mass matrix M_l, we choose M_is of the form $M_i = \text{diag}(\bar{z}, 0, \omega^{i-1} z)$ where z is a complex number with $|z| \ll 1$. After spontaneous symmetry breaking, the resulting $M_l = M_l^0 + \Delta M_l$, where M_l^0 is given in eq. (2.9) and

$$\Delta M_l = \begin{pmatrix} h_1 v \bar{z} & h_2 v \bar{z} & h_3 v \bar{z} \\ 0 & 0 & 0 \\ h_1 v z & h_2 v z \omega & h_3 v z \omega^2 \end{pmatrix}. \tag{3.3}$$

Such a ΔM_l can arise from higher order effects of the theory. The form of ΔM_l is similar to the form of $\Delta M_{u,d}$ given in eq. (4.3) of [35]. In generating these higher order terms, the Higgs VEVs are unaffected. To simplify the phenomenological analysis, we parameterized all the six higher order terms in terms of a single number z. Note that there is no residual symmetry left in the charged lepton sector after the spontaneous symmetry breaking. The perturbed matrix elements of M_l introduce reciprocal factors in the respective columns of U_ω. Requiring U_ω to be unitary, we get z to be

$$z = -1 \pm \sqrt{1 - s^2} + is. \tag{3.4}$$

We will retain the solution with + sign in order to keep $|z| < 1$. The perturbation strength is of the order s which we take to be small. Using the parametrization $s = \sin\alpha$, we can transform U_ω to

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} e^{i\alpha} & 1 & e^{-i\alpha} \\ e^{i\alpha} & \omega & \omega^2 e^{-i\alpha} \\ e^{i\alpha} & \omega^2 & \omega e^{-i\alpha} \end{pmatrix}. \tag{3.5}$$

The PMNS matrix becomes

$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\alpha} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{3.6}$$

A similar structure for the PMNS matrix is discussed in refs. [48–50]. From the above equation, we can compute the perturbed values of the mixing angles to be

$$\begin{aligned}
\sin^2\theta_{13} &= \frac{2}{3}\sin^2\alpha &&= \frac{2s^2}{3} \\
\sin^2\theta_{12} &= \frac{1}{2+\cos 2\alpha} &&= \frac{1}{3}+\frac{2s^2}{9}+O(s^3) \\
\sin^2\theta_{23} &= \frac{2+\cos 2\alpha+\sqrt{3}\sin 2\alpha}{2(2+\cos\, 2\alpha)} &&= \frac{1}{2}+\frac{s}{\sqrt{3}}+O(s^3).
\end{aligned} \tag{3.7}$$

The sine squared values of mixing angles in this scheme are plotted in Figure 1.

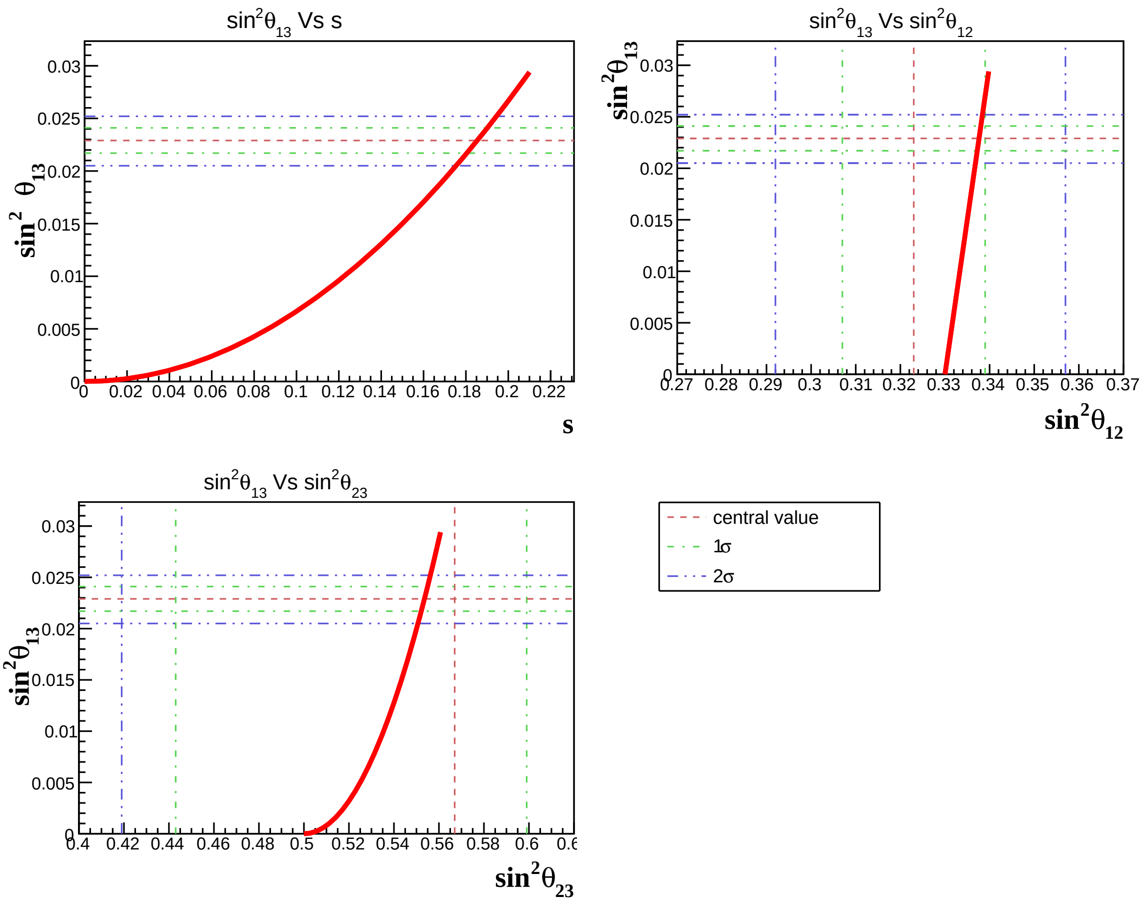

Figure 1: The plot of sine squared values of the mixing angles due to a $Z_2 \times Z_2$ invariant perturbation in the charged lepton sector. Lines demarcating the central values and the 1σ and 2σ allowed regions are shown explicitly.

The perturbation parameter $s \sim 0.19$ leads to a very good fit for θ_{13}. Such a value of s also gives $\sin^2\theta_{23}$ very close to the central value and $\sin^2\theta_{12}$ within 2σ range. Compared to their respective TBM values, $\sin^2\theta_{12}$ changes very little ($\sim 5\%$), whereas $\sin^2\theta_{23}$ receives an appreciable correction ($\sim 14\%$).

We introduced perturbations in both the first and third rows of M_l. We chose these perturbations to be related to each other. This enabled us to keep the perturbation s at the level of $10-20\%$. In principle, it is possible to choose the perturbing matrix $M_i = \text{diag}\,(\bar{z},\, 0,\, 0)$. Such a perturbation modifies only the first row of M_l. Parametrizing z in terms of s as in eq. (3.4), we can obtain the modified values of the mixing angles. With $s = \sin\alpha$, these values are

$$\begin{aligned}
\sin^2\theta_{13} &= \frac{2}{3}\sin^2\frac{\alpha}{2} &&= \frac{s^2}{6} + O(s^4) \\
\sin^2\theta_{12} &= \frac{1}{2+\cos\alpha} &&= \frac{1}{3} + \frac{s^2}{18} + O(s^4) \\
\sin^2\theta_{23} &= \frac{2+\cos\ \alpha + \sqrt{3}\sin\alpha}{2(2+\cos\ \alpha)} &&= \frac{1}{2} + \frac{s}{2\sqrt{3}} + O(s^3).
\end{aligned} \tag{3.8}$$

In this case, the amount of perturbation should be double that of the previous case to obtain an acceptable value of θ_{13}.

Given that we obtained viable values of θ_{13} we check if a CP violating phase δ_{CP} is also generated. However, we find that the Jarlskog invariant J of the PMNS matrix in eq. (3.6) is zero. Hence, no CP violation can be generated by the perturbations considered here. So we look for other possible sources of CP violation and also non-zero θ_{13} in this model.

4 Perturbation in Neutrino Sector

In the previous section it was shown that a $Z_2 \times Z_2$ invariant perturbation in the charged lepton sector can give rise to viable θ_{13} but no CP violation. In this section, we add a similar perturbation in the neutrino sector and study its influence on θ_{13} and δ_{CP}. As in the case of the charged lepton sector, the perturbing matrix should be diagonal to satisfy the $Z_2 \times Z_2$ symmetry. We will derive expressions for θ_{13} and δ_{CP} as a function of the two perturbations and show that it is possible to obtain any value of δ_{CP}. It is shown that perturbation only in the neutrino sector leads to maximal CP violation.

We observe that the diagonalizing matrix in the neutrino sector is a rotation matrix of angle $\pi/4$. A small imbalance in the degeneracy of 11 and 33 elements of M_R in eq. (2.9) shifts the rotation angle slightly away from $\pi/4$ [35]. Such an imbalance can be introduced by a $Z_2 \times Z_2$ invariant perturbation in the neutrino sector. We choose this perturbation to be [35, 48, 49]

$$M \underset{(\underline{\mathbf{3}})}{\nu_{\mathrm{R}}^{\mathrm{T}}} C^{-1} \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -a \end{pmatrix} \underset{(\underline{\mathbf{3}})}{\nu_{\mathrm{R}}}. \tag{4.1}$$

The mass M is a A_4 invariant soft term in the lagrangian. The perturbation in eq. (4.1) can be introduced as an A_4 breaking but $Z_2 \times Z_2$ preserving soft term in the lagrangian.

The perturbed Majorana mass matrix becomes

$$\begin{pmatrix} M+aM & 0 & M' \\ 0 & M & 0 \\ M' & 0 & M-aM \end{pmatrix}. \tag{4.2}$$

This matrix can be diagonalized by a rotation of angle 'x', where $\tan 2x = M'/aM$. We will denote the perturbation in the neutrino sector by the dimensionless parameter $\zeta = aM/M'(\equiv \cot 2x)$. The form of PMNS matrix after the combined perturbations in the charged lepton and the neutrino sectors is

$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\alpha} \end{pmatrix} \begin{pmatrix} \cos\ x & 0 & -\sin\ x \\ 0 & 1 & 0 \\ \sin\ x & 0 & \cos\ x \end{pmatrix}. \tag{4.3}$$

We recall that the perturbation in the charged lepton sector $s = \sin\alpha$. The Jarlskog invariant of this matrix can be found to be $\sqrt{3}\cos 2x/18$ which vanishes for $x = \pi/4$. We obtain CP violation due to the deviation of the angle 'x' from $\pi/4$ through the perturbation in the neutrino sector. Expanding the expressions for the mixing angles up to order ζ^2 and s^2, we get

$$\sin^2\theta_{13} = \frac{1}{3}(1-\cos 2\alpha \sin 2x) = \frac{\zeta^2}{6} + \frac{2}{3}s^2 - \frac{\zeta^2 s^2}{3}, \tag{4.4}$$

$$\sin^2\theta_{12} = \frac{1}{2+\cos 2\alpha \sin 2x} = \frac{1}{3} + \frac{\zeta^2}{18} + \frac{2}{9}s^2 - \frac{\zeta^2 s^2}{27}, \tag{4.5}$$

$$\sin^2\theta_{23} = \frac{2+\cos 2\alpha \sin 2x + \sqrt{3}\sin 2x \sin 2\alpha}{4+2\cos 2\alpha \sin 2x} = \frac{1}{2} + \frac{s}{\sqrt{3}} - \frac{\zeta^2 s}{3\sqrt{3}}. \tag{4.6}$$

From these values and the Jarlskog invariant, we obtain $\sin\delta_{CP}$ to be

$$\sin\ \delta_{CP} = \frac{\cos 2x(2+\cos 2\alpha \sin 2x)}{\sqrt{\left(1-\cos^2 2\alpha \sin^2 2x\right)\left[4+4\cos 2\alpha \sin 2x + (-1+2\cos 4\alpha)\sin^2 2x\right]}}. \tag{4.7}$$

The expression in eq. (4.7) is exact. We can obtain a simpler equation by expanding it in ζ and s and keeping only the leading powers in the numerator and the denominator,

$$\sin\ \delta_{CP} = -\frac{\zeta}{\sqrt{4s^2 + \zeta^2 - \frac{16 s^2 \zeta^2}{3}}}. \tag{4.8}$$

The value of δ_{CP} goes to zero as ζ tends to zero, corresponding to no perturbation in the neutrino sector. For perturbation only in the neutrino sector, we have $s = 0$ and $\delta_{CP} = \pm\pi/2$, depending on the sign of ζ. The ν_e appearance data of T2K prefers δ_{CP} to be in the lower half plane. From eq. (4.7), this indicates that ζ should be positive. The best fit value of this data is equal to -$\pi/2$ which prefers that perturbation in the charged lepton sector is extremely small. The value of δ_{CP} depends on the relative strengths of the perturbations, s in the charged lepton sector and ζ in the neutrino sector. This dependence

is plotted in figure 2. From this figure, we note that $\zeta \geq 2s$ if $\delta_{CP} \geq \pi/4$ and δ_{CP} quickly becomes very small for $\zeta < s$. Figure 3 shows the variation of mixing angles with respect to ζ where the bands for 1σ and 2σ bounds are also drawn. The value of $\sin^2\theta_{13} \approx 0.025$ near $\zeta \approx 0.36$. For this value of ζ, the change in $\sin^2\theta_{12}$ is negligibly small($\sim 3\%$). Eventhough the value of ζ is moderately large, the parameter $a = \zeta M'/M$ quantifying the perturbation in the neutrino sector is quite small because $M' \ll M$. The value of $\sin^2\theta_{23}$ remains 0.5 if the perturbation in the charged lepton sector is zero.

In the present scenario, there is a tension between obtaining a large δ_{CP} and a value of $\sin^2\theta_{23}$ close to the best fit experimental value. This value of $\sin^2\theta_{23}$ is 15% larger than TBM value of 0.5. In order to obtain this large a deviation, we need a value of $s \approx 0.19$ in the charged lepton sector. For $s \approx 0.19$, the constraint on $\sin^2\theta_{13}$ in eq. (4.4) leads to very small values of ζ and hence of δ_{CP}. A large CP violation, on the other hand, requires $\zeta > 2s$, which keeps the value of $\sin^2\theta_{23}$ close to the TBM value of 0.5, as can be seen from eq. (4.6). Current experiments T2K and NOνA can improve the precision on $\sin^2\theta_{23}$. If the central value comes closer to 0.5, then it is possible to have large CP violation. Otherwise, the CP violation is constrained to remain small in this scenario.

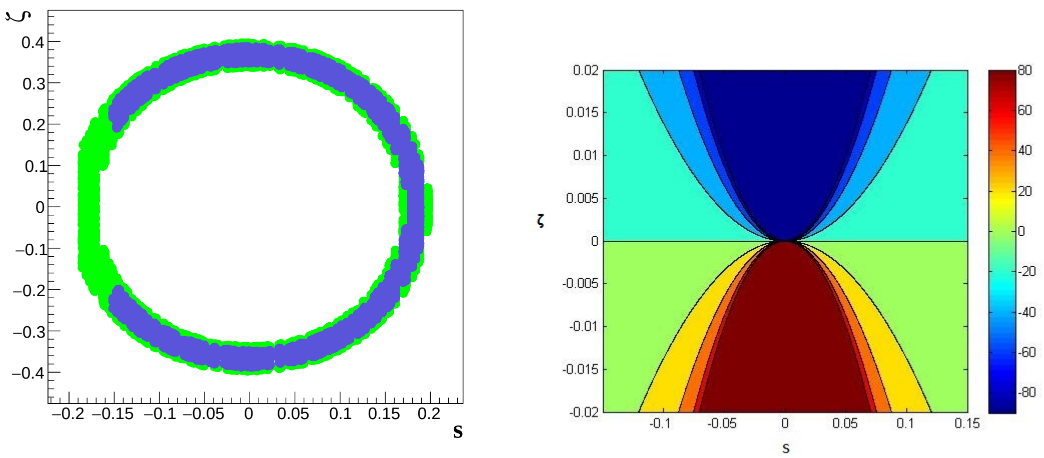

Figure 2: Left panel: The points in $s - \zeta$ space which satisfy the 2σ (blue band) and 3σ (green band) constraints on $\sin^2\theta_{12}$, $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$. Right panel: The value of δ_{CP} for different regions in the $s - \zeta$ space.

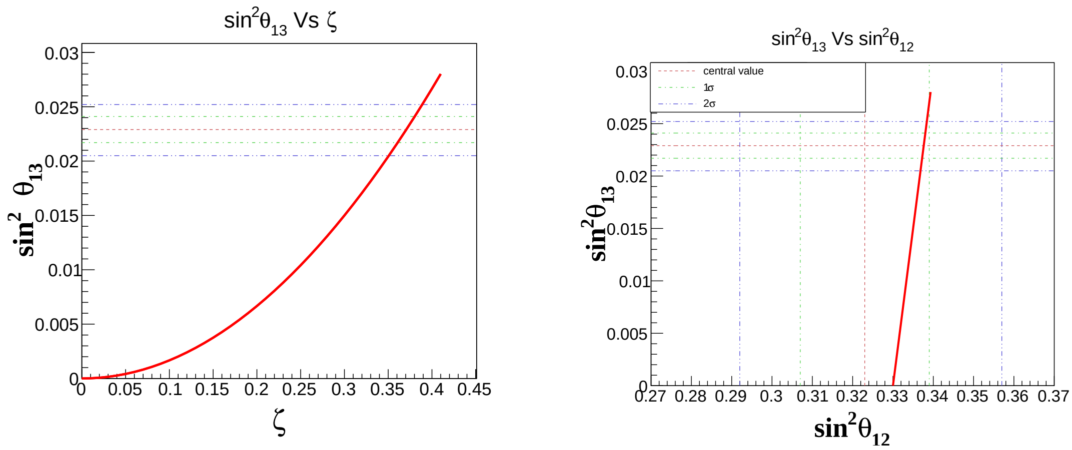

Figure 3: The plot of sine squared vales of mixing angles for maximal δ_{CP} through a $Z_2 \times Z_2$ invariant perturbation with lines for 1σ and 2σ range.

5 Summary and Conclusion

We consider the phenomenology of a model with A_4 symmetry which predicts the tribimaximal form for the PMNS matrix. In this model, we have introduced $Z_2 \times Z_2$ invariant perturbations in both the charged lepton and the neutrino sectors. We find that perturbations in the charged lepton sector alone ($\zeta = 0$) can lead to acceptable values of θ_{13} but do not give any CP violation. But, perturbations purely in the neutrino sector ($s = 0$) give rise to viable values of θ_{13} and maximal CP violation. Any desired value of the CP violating phase δ_{CP} can be obtained by choosing the appropriate values for the perturbations in the charged lepton and neutrino sectors. However, there is a tension between the requirement to obtain a large CP violation and the need to have the value of $\sin^2\theta_{23}$ close to its best fit value. The current experiments may be able to settle this issue. The final Lagrangian has no overall residual symmetry even though the neutrino sector has a residual Z_2 symmetry. It will be interesting to explore whether there could be some consequences due to residual symmetry in neutrino sector.

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